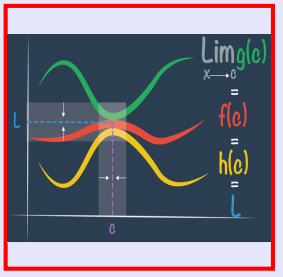
Calculus I

Lecture 44



Feb 19-8:47 AM

More on integration

1) $\int f(x)\,dx = F(x) + C$ where $\frac{d}{dx}[F(x)+C] = f(x)$

2) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$.

3) $\int \cos x\,dx = \sin x + C$

4) $\int \sin x\,dx = -\cos x + C$

5) $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

6) $\int c f(x)\,dx = c\int f(x)\,dx$

7) $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ Definite Integral

Nov 19-7:27 AM

Find $\int (2x + \cos x)\,dx =$

$\int 2x\,dx + \int \cos x\,dx =$

$2\int x^1\,dx + \int \cos x\,dx = 2 \cdot \frac{x^2}{2} + \sin x + C$

$= \boxed{x^2 + \sin x + C}$

Nov 19-7:32 AM

Evaluate $\int_0^1 [3x^2 - 4]\,dx$

$= \int_0^1 3x^2\,dx - \int_0^1 4\,dx$

$= 3\int_0^1 x^2\,dx - 4\int_0^1 dx$

$= \cancel{3} \cdot \frac{x^3}{\cancel{3}}\Big|_0^1 - 4x\Big|_0^1$

$= (x^3 - 4x)\Big|_0^1 = (1^3 - 4(1)) - (0^3 - 4(0))$

$= 1 - 4 - 0$

$= \boxed{-3}$

Nov 19-7:35 AM

Find the area below $f(x) = x + \sqrt{x}$, above x-axis from $x=1$ to $x=4$.

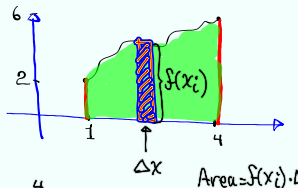
$f(x) = x + \sqrt{x}$

$f(1) = 2$

$f(4) = 6$

Domain $[0, \infty)$

$f(x) \geq 0$



Area $= f(x_i) \cdot \Delta x$

$$A = \int_1^4 f(x)\,dx$$

$$= \int_1^4 \left[x + \sqrt{x}\right] dx$$

$\sqrt{x} = x^{1/2}$

$$= \int_1^4 x\,dx + \int_1^4 \sqrt{x}\,dx$$

$$= \left(\frac{x^2}{2} + \frac{x^{3/2}}{3/2} \right) \Bigg|_1^4$$

$$= \left(\frac{1}{2}x^2 + \frac{2}{3}x\sqrt{x} \right) \Bigg|_1^4 = \left(\frac{1}{2}\cdot 4^2 + \frac{2}{3}\cdot 4\sqrt{4} \right) - \left(\frac{1}{2}\cdot 1^2 + \frac{2}{3}\cdot 1\sqrt{1} \right)$$

$$= 8 + \frac{16}{3} - \frac{1}{2} - \frac{2}{3}$$

$$= \frac{16}{2} + \frac{16}{3} - \frac{1}{2} - \frac{2}{3} = \frac{15}{2} + \frac{14}{3}$$

$$= \frac{15 \cdot 3 + 2 \cdot 14}{6} = \boxed{\frac{73}{6}}$$

Nov 19-7:39 AM

Find abs. min. & abs. max. of

$f(x) = \dfrac{x}{x^2 - x + 1}$ $[0, 3]$

$f(0) = 0$

$f(3) = \dfrac{3}{9 - 3 + 1} = \dfrac{3}{7}$

Is $f(x)$ cont. on $[0,3]$?

$$f'(x) = \frac{1(x^2 - x + 1) - x(2x - 1)}{(x^2 - x + 1)^2} = \frac{x^2 - x + 1 - 2x^2 + x}{(x^2 - x + 1)^2}$$

$$= \frac{1 - x^2}{(x^2 - x + 1)^2}$$

$f'(x) = 0$

$1 - x^2 = 0$

$x = \pm 1 \rightarrow x = 1$

$x = -1$ is not in $[0,3]$

$f(1) = \dfrac{1}{1^2 - 1 + 1} = 1$

$f'(x)$ undef.

$x^2 - x + 1 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

No real solns.

$f'(x)$ defined $(-\infty, \infty)$

$f(x)$ cont. $(-\infty, \infty)$

$f(0) = 0$ ← Min

$f(1) = 1$ ← Max

$f(3) = \dfrac{3}{7}$

Nov 19-7:50 AM

Find all points on $4x^2+y^2=4$ that are farthest from $(1,0)$.

$y^2=4-4x^2$

$y^2=4-4\left(\frac{-1}{3}\right)^2 \rightarrow y=\pm\frac{4\sqrt{2}}{3}$

$y^2=4-\frac{4}{9}=\frac{32}{9}$

$d=\sqrt{(x-1)^2+(y-0)^2}$

$d=\sqrt{(x-1)^2+y^2}$ Max.

$f(x)=(x-1)^2+4-4x^2$

$f'(x)$, $f'(x)=0$, $f''(x)$, $f''(C.P.)$

$f'(x)=2(x-1)^{1}\cdot 1-8x=2x-2-8x=-6x-2$

$f'(x)=0 \rightarrow -6x-2=0 \rightarrow x=\frac{-1}{3}$

$f''(x)=-6<0$ CD Max.

$\frac{x^2}{1}+\frac{y^2}{4}=1$

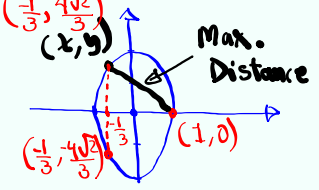
Int. Algebra, Algebra II, College Algebra, Precalc.

Ellipse

$(0,0)$

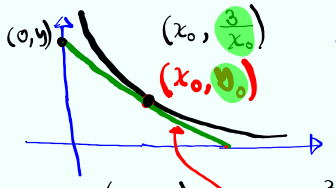
± 1 horizontally

± 2 vertically



Nov 18-8:13 AM

What is the shortest possible length of a line segment in the first quadrant that is tangent to the curve $y=\frac{3}{x}$ at some point?



$y'=\frac{-3}{x^2}$

$m_{tan.line}=y'\big|_{(x_0,y_0)}$

$m=\frac{-3}{x_0^2}$

$y-y_1=m(x-x_1)$

$y-\frac{3}{x_0}=\frac{-3}{x_0^2}(x-x_0)$

$y=\frac{-3}{x_0^2}x+\frac{3x_0}{x_0^2}+\frac{3}{x_0}$

$y=\frac{-3}{x_0^2}x+\frac{3}{x_0}+\frac{3}{x_0}$

$y=\frac{-3}{x_0^2}x+\frac{6}{x_0}$

$0=\frac{-3}{x_0^2}x+\frac{6}{x_0}$

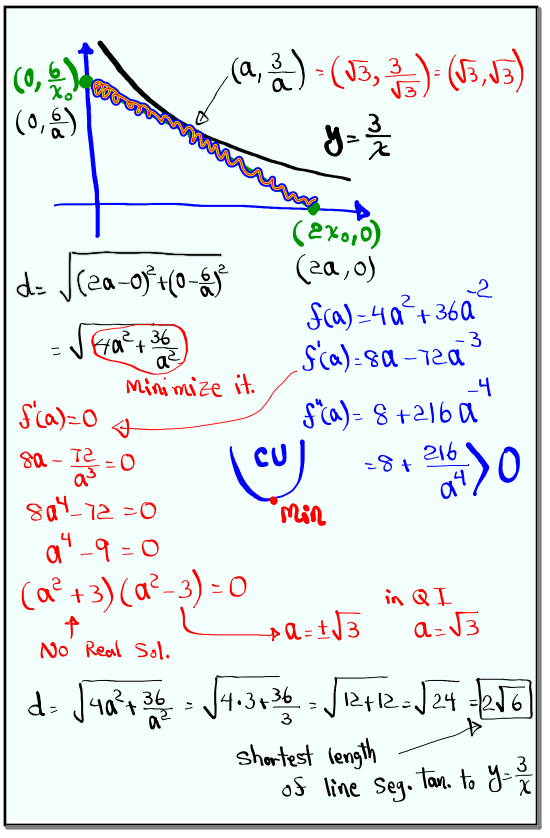
$\frac{3}{x_0^2}x=\frac{6}{x_0}$

$x=\frac{\frac{6}{x_0}}{\frac{3}{x_0^2}}=\frac{6}{x_0}\cdot\frac{x_0^2}{3}=2x_0$

Y-Int $(0,\frac{6}{x_0})$

X-Int $(2x_0,0)$

Nov 18-8:22 AM



Nov 19-8:14 AM

Class QZ 16 (open notes)

Find abs. Max & abs. Min. of

$f(x) = 5 + 54x - 2x^3$ on $[0, 4]$.

$f(x)$ is a polynomial $\Rightarrow$ Cont. & Diff. $(-\infty, \infty)$

$f(0) = 5$, $f(4) = 93$

$f'(x) = 54 - 6x^2$

$f'(x) = 0$

$54 - 6x^2 = 0$

$x^2 = 9$

$x = 3$

$f(3) = 113$

Abs. Max @ $(3, 113)$

Abs. Min @ $(0, 5)$

Abs. Max. Value 113

Abs. Min. Value 5

Nov 19-8:25 AM